2017

Full Marks - 60

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer ***all*** questions selecting either {(a),(b)} or {(c),(d)} from each question

Symbols used have their usual meaning

1. a) Solve $(D^2 + 2D + 1)y = xe^x \sin x$. 8

b) i) Define Cauchy-Euler Equation. Using this method solve

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0.$$ 2

ii) Define Linear differential Equation. Find the Integrating Factor of

$$(1 + y^2)dx = (\tan^{-1}y - x)dy.$$ 2

OR

c) Solve $y^{iv} - 3y''' - 2y'' + 2y' + 12y = 0$. 8

d) i) Find y_p for the Differential Equation

$$(D^2 - 4D + 4)y = x^2.$$ 2

ii) Write the working rules for solving problems of linear equations of second degree by normal form. 2

2. a) Solve

$$\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$$ 8

b) i) Show that

$$(2x + y^2 + 2xz)dx + 2xydy + x^2dz = 0$$

is integrable. 2

ii) Define Pfaffian Differential Equations. Write the Pfaffian Differential Equation in three independent variables. Under what condition the Pfaffian differential equation possess a solution ? 2

OR

ii) Find the two ordinary Linear Differential Equations from Wave Equation by method of Separation of variables using boundary conditions. 2

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ii) Find the elementary solution of

$$u_t - ku_{xx} = 0.$$ 2

OR

c) Solve the following initial boundary value problem for the transient temperature u(x, y, t) for the diffusion of heat in the rectangular plate of uniform, isotropic material : 8

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{k}\frac{\partial u}{\partial t},\ 0 < x < a,\ 0 < y < b, t > 0$$

$$u(x, 0, t) = 0,\ u(x, b, t) = 0,\ 0 < x < a, t > 0$$

$$u(0, y, t) = 0,\ u(a, y, t) = 0,\ 0 < y < b, t > 0$$

$$u(x, y, 0) = f(x, y),\ 0 < x < a,\ 0 < y < b.$$

d) i) Write Laplace's Equation in three dimensional and hence express Laplace's Equation in Ploar co-ordinates and cylindrical Co-ordinates. 2

c) Verify that the equation

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

is integrable and find its primitive. 8

d) i) Solve $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$. 2

ii) Solve $\frac{dx}{y^2} = \frac{dy}{x^2} = \frac{dz}{x^2y^2z^2}$. 2

3. a) Find the geneal integral of the partial differential equation cos (x+y) p + sin (x+y) q = z. 8

b) i) Write the Clairant Equation and hence write the complete integral of

$$z = px + qy - 2\sqrt{pq}.$$ 2

ii) Write the Charpit's subsidiary equation to solve the non-linear first order partial differential equation in the form F(x, y, z, p, q) = 0. 2

OR

[Turn Over

c) Find the complete, singular and general integral of the equation $q - p + x - y = 0$. 8

d) i) Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x + a)(y + b)$. 2

ii) Solve $yzp + zxq = xy$ by Lagrange's Equation method. 2

4. a) Solve

$$\left(x^2D^2 - xyDD' - 2y^2D'^2 + xD - 2yD'\right)z = \log\frac{x}{y}$$

where $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$ 8

b) i) Find particular integral of the equation

$$(D^2 - D')z = e^{x+y}$$ 2

ii) Solve

$$(2D + D' - 1)^2 (D - 2D' + 2)^3 z = 0.$$ 2

OR

c) Solve $(D^2 - DD' - 2D'^2 + 2D' + 2D)z$
$= e^{2x+3y} + \sin(2x + y)$. 8

d) i) Solve $(2D^2 + D'^2 + D)Z = 0$. 2

ii) Find particular integral of the equation

$$(D^2 - D')Z = x \sin y.$$ 2

5. a) Find the steady temperature distribution u(x, y) in the uniform unit square $0 \le x \le 1$; $0 \le y \le 1$, when the edge y = 1, is maintained at the temperature $x(1 - x)$, the other three edges being thermally insulated so that $\frac{\partial u}{\partial n}$ along them. 8

b) i) Write one-dimensional wave equation. Describe the function u(x, t) have two boundary conditions at the ends x = 0 and $x = l$ and two initial conditions at t = 0. 2